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Distorted black holes of the Einstein-Klein-Gordon system

J. Estevez-Delgado[1] and T. Zannias[2]
[1]*Facultad de Ciencias Físico-Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Mich, Mexico*
[2]*Instituto de Física y Matemáticas, Universidad Michoacana de SNH, Apartado Postal 2-82, 58040 Morelia, Mich, Mexico*
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We present two families of solutions (g, Φ) of the Einstein-Klein-Gordon system minimally coupled to gravity, massless scalar field equations possessing a regular $\mathbb{R} \times S^2$ bifurcating Killing horizon. The solutions may be interpreted as describing the metric g and the field Φ in an open vicinity containing the event horizon of a Schwarzschild black hole interacting with an exterior, static-axisymmetric distribution of matter carrying scalar charges generating Φ. One family of the solution possesses a spherical bifurcation two sphere and may be interpreted as generated by a distribution of matter carrying scalar charges but of negligible self-gravity. The other family describes an axisymmetric horizon with the distortion caused by the self-gravity of the external matter. For both classes of solutions the scalar field Φ relaxes into a configuration so that its energy-momentum tensor $T^{\mu\nu}$ on the Killing horizon obeys the property: $T^{\mu\nu} l_\mu l_\nu = 0$ where l^μ is a null vector normal to the horizon. It is also indicated that the Einstein-Klein-Gordon system ought to admit distorted black holes characterized by an $\mathbb{R} \times S^1 \times S^1$ event horizon.

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I. INTRODUCTION

Black hole spacetimes are spacetimes possessing remarkable properties. The four laws of black hole mechanics and their relation to the familiar thermodynamical ones suggest a deep connection between gravity, quantum physics, and thermodynamics—a connection that despite persistent efforts still has not been fully understood [1]. If the Cosmic Censorship Hypothesis [2] holds true, then the black hole uniqueness theorems for the vacuum or electrovacuum Einstein's equations [3] imply that the final equilibrium state resulting from the complete gravitational collapse of any bounded system or the end state of a black hole-neutron star pair, or a black hole pair [4], would be described by the Kerr-Neumann family of black holes. Remarkably irrespective of the number of parameters describing the initial state only the mass M, charge Q, and angular momentum J would be registered by the final black hole equilibrium state. Black hole spacetime is not an exclusive property of the vacuo or electrovacuo Einstein's equations. Einstein's gravity coupled to scalar or other fields admits the so-called hairy family of black holes. However, here matters are not yet on a firm state as for the vacuum or electrovacuum case. Even though a number of uniqueness theorems for particular field configurations have been established [5], their classification is for the moment open and mathematically challenging problems are likely to persist for some time [6]. The absence of black hole states within the Einstein-Maxwell system possessing an ergoregion disconnected from the event horizon has been settled only relatively recently with the work of Sudarsky and Wald [7].

Despite this incomplete picture regarding the status of (isolated) black hole equilibrium states, it has been recognized in fact long ago that there exists an infinity parameter family of distorted black holes, i.e., spacetimes representing a central black hole interacting with an external distribution of matter. Israel [8] some time ago (see also the important work of Ref. [9]) analyzed the tidal distortion of the event horizon of a Schwarzschild black hole caused by a strongly gravitating distribution of axisymmetric matter. It was shown in [8] that a vicinity of the distorted horizon can be described by particular axisymmetric solutions of the vacuum equations. The local and global structure of distorted axisymmetric black hole spacetimes have been addressed by Geroch and Hartle [10]. Under the assumption that the event horizon admits compact, connected cross sections, it was shown in [10] that distorted black hole spacetimes fall into two classes. The first class includes spacetimes possessing an $\mathbb{R} \times S^2$ event horizon, while the second family includes spacetimes possessing an $\mathbb{R} \times S \times S$ event horizon, referred to as toroidal black holes. In view of the Geroch-Hartle analysis, the vacuum toroidal black hole constructed by Peters [11] turns out to be only a particular solution. Xanthopoulos [12], building upon earlier work by Chandrasekhar [13], constructed a more general class of axially symmetric distorted by external matter toroidal black holes. Besides the vacuum distorted holes constructed in Refs. [8,10,11], distorted black hole spacetimes, as far as we are aware, have been constructed only for the Einstein-Maxwell system in Ref. [14] and for the Einstein-dilaton gravity in Ref. [15]. We may add here that within the Ashtekar framework of isolated horizons [16], distorted black holes may be viewed as particular solutions of the Einstein's equations admitting isolated horizons. Besides the above mentioned families of dis-

*Electronic address: joaquin@fismat.umich.mx
†Electronic address: zannias@ginette.ifm.umich.mx

torted holes, in Ref. [17], a different type of a distorted black hole spacetime has been constructed. It is a non-asymptotically flat spacetime admitting a Killing horizon which however has been "distorted" by changing the topology of a Schwarzschild hole rather via the effect of an external distribution of matter.

In all of the above mentioned work on distorted black holes, except that of Refs. [14,15], it has been assumed that the perturbing external matter is free of any charges so that the spacetime region between event horizon and perturbing matter is free of any field configurations. In this work we shall assume that the external matter is endowed with a scalar charge distribution so that the same spacetime region is permeated by a massless scalar field Φ obeying the Einstein-Klein-Gordon equations. We shall show that such a region can be described by a spacetime (M, g, Φ) with (g, Φ) particular static-axisymmetric solutions of the Einstein-Klein-Gordon system admitting an isometric extension into a larger manifold (M', g', Φ') possessing a regular $\mathbb{R} \times S^2$ bifurcating Killing horizon [18]. Under suitable extensions discussed in Sec. IV, this bifurcating regular Killing horizon may be elevated to the status of an event horizon and thus (M', g', Φ') may be interpreted as representing the spacetime near the event horizon of a distorted black hole of the Einstein-Klein-Gordon system. In this work we construct two families of such solutions characterized by an infinite set of parameters interpreted as describing the structure of the perturbing distribution of matter as well as the structure of the scalar charge distribution responsible for the field Φ.

II. STATIC-AXISYMMETRIC SOLUTIONS OF THE EINSTEIN-KLEIN-GORDON EQUATIONS, ADMITTING A BIFURCATING KILLING HORIZON

We recall that the Einstein-Klein-Gordon minimally coupled to gravity field equations on a spacetime M are described by

$$G_{\mu\nu} = k(\nabla_\mu \Phi \nabla_\nu \Phi - \tfrac{1}{2} g_{\mu\nu} \nabla^\sigma \Phi \nabla_\sigma \Phi), \quad (1)$$

$$\nabla^\mu \nabla_\mu \Phi = 0. \quad (2)$$

For any nonsingular, minimally C^2 solution (g, Φ) of the above equations admitting two hypersurfaces orthogonal, commuting, and orthogonal Killing vector fields, a timelike one ξ_t and a spacelike ξ_φ chart (t, φ, x^1, x^2) can be constructed [19] so that $\xi_t = \frac{\partial}{\partial t}$, $\xi_\varphi = \frac{\partial}{\partial \varphi}$, and g takes the Weyl canonical form:

$$g = -e^{2U} dt^2 + r^2 e^{-2U} d\varphi^2 + e^{2(V-U)}(dr^2 + dz^2), \quad (3)$$

where $U = U(r, z)$, $V = V(r, z)$, and points on the manifold where $r = 0$ define the symmetry axis associated with the rotational Killing field ξ_φ [20]. Relative to this local chart, the system (1) and (2) implies that $U(r, z)$, $\Phi(r, z)$, and $V(r, z)$ satisfy

$$\frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial z^2} + \frac{1}{r}\frac{\partial U}{\partial r} = 0, \quad (4)$$

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} = 0, \quad (5)$$

$$\frac{\partial V}{\partial r} = r\left\{\left(\frac{\partial U}{\partial r}\right)^2 - \left(\frac{\partial U}{\partial z}\right)^2 + \frac{k}{2}\left[\left(\frac{\partial \Phi}{\partial r}\right)^2 - \left(\frac{\partial \Phi}{\partial z}\right)^2\right]\right\}, \quad (6)$$

$$\frac{\partial V}{\partial z} = r\left[2\frac{\partial U}{\partial r}\frac{\partial U}{\partial z} + k\frac{\partial \Phi}{\partial r}\frac{\partial \Phi}{\partial z}\right]. \quad (7)$$

Equations (4) and (5) are recognized as the Laplace equations on Euclidean $\mathbb{R}^3$ equipped with cylindrical coordinates (r, φ, z) and once a choice of the harmonic functions (U, Φ) has been made, Eqs. (6) and (7) determine V via quadratures. The integrability conditions for the existence of V are satisfied by virtue of (4) and (5).

Our goal is to construct solutions of (4)–(7) so that the resulting (g, Φ) admits an extension possessing a regular bifurcating Killing horizon. In order to identify such solution we consider an open subset $S = \{(r, z, \varphi) | r < a, z < b, ab \neq 0\}$ containing the origin of the Euclidean three space, where (r, z, φ) are standard cylindrical coordinates and (a, b) arbitrary for the moment parameters. We then consider the product manifold $M = \mathbb{R} \times S$ and for any triplet (U, V, Φ) satisfying (4)–(7) on S, we take their lifts on $M = \mathbb{R} \times S$. In view of (3) they define a static-axisymmetric metric g and a scalar field Φ on $M = \mathbb{R} \times S$, satisfying the covariant Eqs. (1) and (2). At first we require that the solutions (U, V, Φ) of (4)–(7) must be chosen so that the resulting (g, Φ) on $M = \mathbb{R} \times S$ would satisfy the following:

(α) elementary flatness holds true on any point of the axis;

(β) the spacetime $(\mathbb{R} \times S, g, \Phi)$ is singularity free.

A point of departure for the specification of such a triplet is the observation that any regular axisymmetric harmonic functions, say, standing for field Φ on S can be represented in the form:

$$\Phi(r, z) := \sum_{l=0}^{\infty} \alpha_l (r^2 + z^2)^{l/2} P_l\left(\frac{z}{\sqrt{r^2 + z^2}}\right), \quad (8)$$

where α_l, $l = 0, 1, \ldots$ are arbitrary constants while P_l stands for the Legendre polynomials. We begin applying the above procedure by taking Φ as above and choosing for U the trivial harmonic function $U \equiv 0$. Despite this special choice, the function V resulting from the integration of (6) and (7) is rather complicated. For simplicity we shall employ hereafter a truncated version of (8) described by

$$\Phi(r, z) = \alpha_0 + \alpha_1 z + \alpha_2 (2z^2 - r^2). \quad (9)$$

Making use of this $\Phi(r, z)$, the integration of (6) and (7) combined with $U \equiv 0$ yields

$$V(r, z) = \tfrac{1}{4}kr^2[2\alpha_2^2 r^2 - (4\alpha_2 z + \alpha_1)^2] + V_0. \quad (10)$$

By setting $V_0 = 0$ it follows immediately that $V(r, z)$ is vanishing on the entire z axis. The so-constructed $V(r, z)$ combined with $U(r, z) \equiv 0$ defines on $M = \mathbb{R} \times S$ the metric

$$g = -dt^2 + r^2 d\varphi^2 + e^{2V(r,z)}(dr^2 + dz^2), \quad (11)$$

which is a static, axially symmetric metric admitting $\frac{\partial}{\partial t}$ as a timelike Killing vector field possessing complete orbits, commuting with the axial Killing field $\frac{\partial}{\partial \varphi}$, and both fields are hypersurface orthogonal. By virtue of the fact that $V(r = 0, z) = 0$ it is regular on the axis and moreover it can be checked directly that this g combined with Φ described by (9) satisfies the covariant Eqs. (1) and (2). A computation of the scalar invariants $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$, $R^{\alpha\beta}R_{\alpha\beta}$, and R yields [21]

$$R^{\mu\nu\sigma\tau}R_{\mu\nu\sigma\tau} = 4C^{\mu\nu\sigma\tau}C_{\mu\nu\sigma\tau} = 3R^{\mu\nu}R_{\mu\nu} = 3R^2, \quad (12)$$

$$R = [4\alpha_2^2(r^2 + 4z^2) + 8\alpha_1\alpha_2 z + \alpha_1^2]e^{f(r,z)}, \quad (13)$$

$$f(r, z) = \tfrac{1}{2}kr^2[2\alpha_2^2(8z^2 - r^2) + \alpha_1(8\alpha_2 z + \alpha_1)],$$

indicating a regular geometry on any point on $M = \mathbb{R} \times S$. Extending the coordinates (r, z) over the entire plane, the resulting spacetime $(M = \mathbb{R} \times \mathbb{R}^3, g, \Phi)$ fails to be asymptotically flat and thus (9)–(11) appear to be of limited utility. However, by appealing to the partial linearity afforded by (4)–(7), they can used as a seed for the construction of more interesting families of static-axisymmetric solutions of (1) and (2). In that regard we recall that the positive mass Schwarzschild metric in Weyl coordinates (3) is described by

$$U_{BH} = \frac{1}{2}\ln\left(\frac{R_+ + R_- - 2m}{R_+ + R_- + 2m}\right),$$
$$V_{BH} = \frac{1}{2}\ln\frac{(R_+ + R_-)^2 - 4m^2}{4R_+R_-}, \quad (14)$$
$$R_\pm^2 = r^2 + (z \pm m)^2,$$

which are well defined on the entire (r, z) plane except for points lying on $r = 0, z \in [-m, m]$. An application of the Gauss theorem or by appealing to Eqs. (6) and (7) shows that the logarithmic potential U_{BH} satisfies

$$\lim_{r\to 0} r\frac{\partial U_{BH}}{\partial r} = 1, \qquad \lim_{r\to 0} r\frac{\partial U_{BH}}{\partial z} = 0, \qquad -m < z < m. \quad (15)$$

Working on the modified domain $\overline{S} = S - \{-m \le z \le m\}$, and utilizing the pair (Φ, U_{BH}), we construct the function V via integration of Eqs. (6) and (7) on the part of the (r, z) plane with the line segment $(r = 0, -m \le z \le m)$ deleted. Going through the integration procedure we find

$$V(r, z) = V_{BH}(r, z) + V_\Phi(r, z)$$
$$= \frac{1}{2}\ln\left(\frac{(R_+ + R_-)^2 - 4m^2}{4R_+R_-}\right) + \frac{1}{4}kr^2 \times\left[2\alpha_2^2 r^2 - (4\alpha_2 z + \alpha_1)^2\right], \quad (16)$$

where for latter convenience we have split V into contributions V_{BH} and V_Φ arising, respectively, from U_{BH} and Φ. It can be easily checked that this $V(r, z)$ is regular over the two disconnected components of the symmetry axis and combined with U_{BH} generates the following metric on $M = \mathbb{R} \times \overline{S}$:

$$g = -e^{2U_{BH}}dt^2 + r^2 e^{-2U_{BH}}d\varphi^2 + e^{2V-2U_{BH}}(dr^2 + dz^2). \quad (17)$$

Transforming to new spherical-like coordinates (r_s, θ) via $r^2 = r_s(r_s - 2m)\sin^2\theta$, $z = (r_s - m)\cos\theta$ we obtain

$$g = -\left(1 - \frac{2m}{r}\right)dt^2 + e^{2V_\Phi}\left(1 - \frac{2m}{r}\right)^{-1}dr^2 + r^2 e^{2V_\Phi}(d\theta^2 + e^{-2V_\Phi}\sin^2\theta d\varphi^2), \quad (18)$$
$$r \in (2m, r_b),$$

$$V_\Phi(r, \theta) = \tfrac{1}{4}kr(r - 2m)\sin^2\theta\{2\alpha_2^2 r(r - 2m)\sin^2\theta - [4\alpha_2(r - m)\cos\theta + \alpha_1]^2\}, \quad (19)$$

$$\Phi(r, \theta) = \alpha_0 + \alpha_1(r - m)\cos\theta + \alpha_2[2(r - m)^2\cos^2\theta - r(r - 2m)\sin^2\theta]$$
$$= \alpha_0 + \frac{2}{3}\alpha_2 m^2 + \alpha_1(r - m)\cos\theta + \frac{\alpha_2}{3}[3(r - m)^2 - m^2](3\cos^2\theta - 1), \quad (20)$$

where in the above and for notational simplicity r_s has been replaced everywhere by r and the passage to the second line in (20) has been done by grouping together the various terms so that $\Phi(r, \theta)$ exhibits clearly its (Newtonian) multiple structure. In the region covered by the local chart (18) the geometry is static and axially symmetric. Moreover, this region is free of any kind of curvature singularities. For instance, the scalar curvature R of (18) is given by

$$R = \frac{k}{r^2}[(r-m)^2 - m^2\cos^2\theta]\{4\alpha_2^2[3(r-m)^2 + m^2]\cos^2\theta + 8\alpha_1\alpha_2(r-m)\cos\theta + 4\alpha_2^2 r^2 - 8\alpha_2^2 r + \alpha_1^2\}e^{f(r,\theta)},$$
$$f(r,\theta) = \frac{k}{2}r(r-2m)\sin^2\theta[2\alpha_2^2(3r-2m)(3r-4m)\cos^2\theta + 8\alpha_1\alpha_2(r-m)\cos\theta - 2\alpha_2^2 r^2 + 4m\alpha_2^2 + \alpha_1^2], \tag{21}$$

and the fact it is well behaved as $r \to 2m$ indicates but does not prove that (18)–(20) ought to admit an extension through $r = 2m$. We shall explicitly construct such extension but before we do so we shall construct another class of solutions of the system (1) and (2). The solutions (g, Φ) described by (18)–(20) can be thought as a "superposition" of the solutions generated by $(U \equiv 0, \Phi)$ and $(U_{BH}, \Phi = 0)$. However, we are free to consider a more general superposition by choosing any regular axisymmetric harmonic function U_m defined on the domain $S \subset \mathbb{R}^3$. Making use of this freedom we consider the combination, $U = U_{BH} + U_m$, defined on $\overline{S} = S \setminus \{-m \leq z \leq m\}$ and for simplicity the scalar field Φ would be still described by (9). Regularity of V along the two disconnected components of the axis places restriction upon the structure of U_m. At first by an analysis of Eqs. (6) and (7) in view of (15), it shows that $V(r=0, z) = 0$ on both parts of the axis, provided the harmonic function U_m representing the external matter satisfies

$$U_m(0, -m) = U_m(0, m). \tag{22}$$

Suppose now that U_m satisfies such condition. "Splitting" V into

$$V = V_{BH} + V_\Phi + \delta, \tag{23}$$

where δ is the part of V generated by the interaction term arising from U_m and U_{BH} in (6) and (7). By utilizing (23) and Eqs. (6) and (7), combined with regularity of Φ and U_m on S and relations (15), it follows that

$$\delta - 2U_m = -2U(0, m) \equiv -2U_0, \qquad r = 0, \quad -m \leq z \leq m, \tag{24}$$

a relation interpreted as a necessary condition for the black hole to be in equilibrium with the external matter field [8]. Assuming that U_m satisfies (22), then the $V = V_{BH} + V_\Phi + \delta$ would be regular over the entire z axis.

For any choice U_m consistent with (22) we define metric g on $\mathbb{R} \times \overline{S}$ by

$$g = -e^{2(U_{BH}+U_m)}dt^2 + r^2 e^{-2(U_{BH}+U_m)}d\varphi^2 + e^{2(V_{BH}+V_\Phi+\delta-U_{BH}-U_m)}(dr^2 + dz^2), \tag{25}$$

where in the above the functions V_{BH} and V_Φ are described by (16) while δ is implicitly determined once a choice of U_m has been made. The spacetime $(M = \mathbb{R} \times \overline{S}, g, \Phi)$ with $\Phi(r, z)$ defined in (9) represents another candidate solution of the Einstein-Klein-Gordon system admitting a regular Killing horizon. Passing to spherical-like coordinates (r_s, θ) via $r^2 = e^{2U_0}r_s(r_s - m_0)\sin^2\theta$, $z = e^{U_0}(r_s - m)\cos\theta$, $m = e^{U_0}m_0$, it follows that (25) is transformed into

$$g = -\left(1 - \frac{2m_0}{r}\right)e^{2(U_m-U_0)}dt^2 + e^{2(V_\Phi+\delta-U_m+U_0)}\left[\left(1 - \frac{2m_0}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + e^{-2(V_\Phi+\delta)}\sin^2\theta d\varphi^2)\right], \tag{26}$$

where again in the above, r_s has been replaced by r and the fields $V_\Phi(r, \theta)$ and $\Phi(r, \theta)$ are given by

$$V_\Phi(r,\theta) = \frac{k}{4}e^{2U_0}r(r-2m_0)\sin^2\theta\{2\alpha_2^2 e^{2U_0}r(r-2m_0)\times\sin^2\theta - [4\alpha_2 e^{U_0}(r-m_0)\cos\theta + \alpha_1]^2\}, \tag{27}$$

$$\Phi(r,\theta) = \alpha_0 + \alpha_1 e^{U_0}(r-m_0)\cos\theta + \alpha_2[2e^{2U_0}(r-m_0)^2\cos^2\theta - e^{2U_0}r(r-2m_0)\sin^2\theta]. \tag{28}$$

A tedious analysis of the long and unyielding scalar invariants associated with (26) shows that as long as (22) holds true then g is singularity free. It is also clear that in the limit $U_m = 0$, naturally (26)–(28) are reduced to (18)–(20).

III. PROPERTIES OF THE GEOMETRY NEAR THE HORIZON

In this section we shall show that the spacetime $(M = \mathbb{R} \times \overline{S}, g, \Phi)$ with g described by (18) or by (26) admits an isometric extension (M', g', Φ') so that g' admits a bifurcating, regular Killing horizon and (g', Φ') satisfy Eqs. (1) and (2) in M'. Since in the limit $U_m \equiv 0$ the metric (26) reduces to (18), we shall devote our attention into the extendibility of the former class. The singularity in the components of g as $r \to 2m_0$ is a mere coordinate singularity, and in fact g can be extended through. We perform this extension by employing a Kruskal-Szekeres chart and relative to this chart the part of the (r, t) plane, $-\infty < t < \infty$, $r \in (2m_0, r_b)$ is mapped into

$$R(r,t) = 4m_0\left(\frac{r}{2m_0} - 1\right)^{1/2}e^{(1/2)[(r/2m_0)-1]}\cosh\left(\frac{t}{4m_0}\right) = F(r)\cosh\left(\frac{t}{4m_0}\right), \tag{29}$$

$$T(r,t) = 4m_0\left(\frac{r}{2m_0} - 1\right)^{1/2} e^{(1/2)[(r/2m_0)-1]}\sinh\left(\frac{t}{4m_0}\right) = F(r)\sinh\left(\frac{t}{4m_0}\right), \tag{30}$$

where $m_0 = me^{-U_0}$. Under this transformation, (26) takes the form:

$$g = -16m_0^2\left(1 - \frac{2m_0}{r}\right)\frac{e^{2(U_m-U_0)}}{F^4(r)}(RdT - TdR)^2 + e^{2(V_\Phi+\delta-U_m+U_0)}\left[\left(1-\frac{2m_0}{r}\right)^{-2}\frac{(RdR - TdT)^2}{F^2(r)(\frac{\partial F}{\partial r})^2} + r^2(d\theta^2 + e^{-2(U_\Phi+\delta)}\sin^2\theta d\varphi^2)\right], \tag{31}$$

which is well defined on points on the manifold subject to $r > 2m_0$. Since on the other hand $R^2 - T^2 = F^2(r)$, it can be written into the equivalent form:

$$g = 2m_0 e^{2(U_m-U_0)}e^{-(r/2m_0)+1}[-dT^2 + dR^2] + e^{2(V_\Phi+\delta-U_m+U_0)}r^2(d\theta^2 + e^{-2(U_m+\delta)}\sin^2\theta d\varphi^2) + \frac{1 - e^{2(V_\Phi+\delta-2U_m+2U_0)}}{4(r-2m_0)}e^{2(U_m-U_0)}(RdR - TdT)^2, \tag{32}$$

and in this form the only components of g where regularity as $r \to 2m_0$ is not manifest is due to the last term. However, regularity is guaranteed by virtue of the properties of the function V_Φ defined by (27) and the fact that the harmonic function U_m is not arbitrary. It suffices to note that $V_\Phi(r,\theta)$ is a smooth function of its arguments and moreover uniformly: $\lim_{r\to 2m_0} V_\Phi(r,\theta) = 0$. This property combined with the condition (24) implies that the last term is regular at $r = 2m_0$ and thus the components of g are regular at $r = 2m_0$. Regularity of (32) at $r = 2m_0$ allows us to extend the range of (R, T) coordinates consistently with $R^2 - T^2 = F^2(r)$ and the constraint $r > 2m - \epsilon$, where here $\epsilon > 0$ [22]. Over the so-extended domain, we extend Φ and the functions (U_m, δ, V_Φ) as analytic functions of (r,θ) for all $r \in (2m_0 - \epsilon, 2m_0 + r_b)$, and thus (32) is well defined for all $r \in (2m - \epsilon, 2m + r_b)$. Moreover, the timelike Killing vector field ξ_t relative to (T, R) coordinates is described by

$$\xi_t = \frac{1}{4m_0}\left[R\frac{\partial}{\partial T} + T\frac{\partial}{\partial R}\right], \qquad r > 2m_0, \tag{33}$$

and we extend it as an analytic function of (T, R) over the entire domain of the Kruskal-Szekeres chart. The so-extended field is well defined and remains a Killing field for the extended g. Via the above extension process, (g, Φ) remains a solution of the Einstein-Klein-Gordon equations, admitting (33) as a Killing vector field. Introducing null coordinates (U, V) defined via

$$T = \tfrac{1}{2}(U + V), \qquad R = \tfrac{1}{2}(V - U), \tag{34}$$

then (31) can be written in the form:

$$g = 2m_0 e^{2(U_m-U_0)}e^{-(r/2m_0)+1}dUdV + e^{2(V_\Phi+\delta-U_\Phi+U_0)}r^2(d\theta^2 + e^{-2(U_\Phi+\delta)}\sin^2\theta d\varphi^2) + \frac{1 - e^{2(V_\Phi+\delta-2U_m+2U_0)}}{16(r-2_{m_0})}e^{2(U_m-U_0)}(UdV + VdU)^2, \tag{35}$$

while the Killing vector field ξ_t is now described by

$$\xi_t = \frac{1}{4m_0}\left[V\frac{\partial}{\partial V} - U\frac{\partial}{\partial U}\right]. \tag{36}$$

In view of the fact $UV = -F(r)$, it follows that ξ_t becomes null at $V = 0$ or $U = 0$ and vanishes identically on $V = U = 0$. Thus in the extended manifold, $r = 2m_0$ is described by the set of points defined either by $V = 0$ or $U = 0$ or contains points lying at the bifurcation sphere $V = U = 0$. The normal to the $U = 0$, respectively, $V = 0$ null hypersurface is parallel to ξ_t and thus it generates a bifurcating-type Killing horizon consisting of two branches, defined by $U = 0$, respectively, $V = 0$ intersecting each other on a two sphere $U = V = 0$. The surface gravity k of the two branches of the horizons can be computed via [20] $k^2 = -\frac{1}{2}\lim_{r\to 2m_0}\nabla^\mu\xi^\nu\nabla_\mu\xi_\nu$ and a long but straightforward calculation in the gauge of (26) taking into account that $\lim_{r\to 2m}V_\Phi(r,\theta) = 0$ combined with (24), shows that $k = 1/4m_0$, implying that the Killing horizon is bifurcate and nondegenerate [23].

Let us now consider the behavior of the field Φ on the Killing horizon. At first $\Phi(r,\theta)$ as defined by (28) is a smooth function of its argument, implying that the stress $T_{\mu\nu}$ obeys

$$\xi^\mu\xi^\nu T_{\mu\nu} = -\tfrac{1}{2}g(\xi_t,\xi_t)\nabla^\mu\Phi\nabla_\mu\Phi, \qquad r > 2m_0.$$

This expression is well defined and smooth over the entire domain of validity of the Kruskal-Szekeres chart. Indeed by the virtue of the fact $G(R,T,r) = R^2 - T^2 - F^2(r) = 0$, $\forall r \in (2m_0 - \epsilon, 2m_0 + r_b)$, and $\partial G/\partial r|_{2m_0} \neq 0$, the implicit function theorem implies that in the local vicinity of $r = 2m_0$, the equation $G(R,T,r) = 0$ can be solved locally for some smooth function $r = r(R,T)$. Moreover, for any (T_0, R_0) so that $G(T_0, R_0, r = 2m_0) = 0$ we have

$$\left.\frac{\partial r}{\partial R}\right|_{2m_0} = \frac{R_0}{4m_0}, \qquad \left.\frac{\partial r}{\partial T}\right|_{2m_0} = -\frac{T_0}{4m_0}.$$

Since in terms of (T, R) coordinates $\Phi = \Phi[r(T,R),\theta]$, the gradient of Φ is well defined. This property combined with the smoothness of g and the fact that ξ_t becomes null over $r = 2m_0$, shows that $\xi^\mu\xi^\nu T_{\mu\nu}| \equiv 0$ on both branches of the horizon and thus the field Φ finds itself in a weightless state over the horizon. This property may be interpreted as the underlying reason that the field Φ coexists peacefully with the horizon. Incidentally, the fact that the gradient of $\Phi[r(T,R),\theta]$ is smooth shows that the components of the stress tensor $T_{\mu\nu}$ expressed in terms of (T,R) coordinates are well defined over the entire Kruskal-Szekeres chart. On the other hand, as a consequence of (35), the area of the bifurcation two sphere $U = V = 0$ is given by

$$\begin{aligned} A &= \int \sqrt{{}^{(2)}g}d\theta d\varphi \\ &= \int_0^{\pi}\int_0^{2\pi}(16m_0^4)^{1/2}\sin^2\theta d\theta d\varphi \\ &= 16\pi m_0^2 = 16\pi m^2 e^{-2U_0}. \end{aligned} \tag{37}$$

Even though the above analysis shows extendibility of the metric (26) and the solution (g,Φ) across $r = 2m_0$, the same analysis can be applied for the other family of solutions described by (18)–(20). In fact, setting $m_0 = m$, $U_m = U_0 = 0$ in (29) and (30) then is a matter of straightforward algebra to verify that identical conclusions hold true for that family of solutions as well. In summary, the family of solutions (g,Φ) described by (18)–(20) and (26)–(28) describe solutions of the Einstein-Klein-Gordon system admitting a regular bifurcating Killing horizon. We shall indicate that under appropriate conditions, those solutions may be extended further so that their extended counterparts represent families of distorted black holes of the Einstein-Klein-Gordon system. Here, however, a different sort of extension is required [10]. As we have mentioned in the previous sections, extending $\overline{S}$, say, over the entire $\mathbb{R}^3$ and extending the functions (Φ, U_m) as solutions of (4) and (5) over the entire $\mathbb{R}^3$, lead to a solution (g,Φ) of the Einstein-Klein-Gordon system that fails to be an asymptotically flat spacetime. Such behavior can be avoided by breaking the harmonicity of Φ and U_m. It is at this point where the effects of the external distribution of matter come into play. Once they are taken into account then Φ and U_m would not any longer satisfy the Laplace equation. The details of how the inclusion of external matter will achieve asymptotic flatness and why the resulting spacetime describes a distorted black hole of the Einstein-Klein-Gordon system is identical to the arguments by Geroch and Hartle in Ref. [10]. Therefore, always having in mind the presence of external matter, the two families of static-axisymmetric solutions (g,Φ) of the Einstein-Klein-Gordon system and their explicit extensions through the Killing horizon discussed above may be equally well interpreted as representing an open vicinity of a distorted black hole of the Einstein-Klein-Gordon system. The fact that those holes possess a nontrivial field, in sharp contrast to some uniqueness theorems regarding the structure of the black holes of the Einstein-Klein-Gordon system [24], is due to the presence of sources outside the horizon.

IV. DISCUSSION

In the present work two families of solutions of the Einstein-Klein-Gordon system admitting a regular bifurcating $R \times S^2$ horizon have been presented. Even though those solutions have been generated using the field Φ described by (9), their extensions for more general Φ can be easily worked out. It is perhaps worth putting the results of the present work in wider perspective. Long ago, Mysak and Szekeres [9] have shown that arbitrary static and strongly gravitating axisymmetric perturbations having support exterior to the horizon of a Schwarzschild black hole remain regular over the perturbed horizon. From the perspective of the present work, Mysak and Szekeres have shown that Einstein's vacuum equations admit solutions g possessing regular $\mathbb{R} \times S^2$ bifurcating Killing horizons. Their conclusion combined with our results and the results obtained in Refs. [14,15] suggest that static-axisymmetric perturbations induced by arbitrary fields are likely to remain regular over a perturbed horizon. A verification of such contention will be of some interest, and in the terminology of the present work, calls for an understanding of the structure and properties of solutions of Einstein's nonvacuum equations admitting regular bifurcating $\mathbb{R} \times S^2$ Killing horizons. Such an investigation is worthwhile to pursue for two independent reasons. At first it would offer additional insights into the dynamics of the Einstein nonvacuum equations and secondly, due to the fact that bifurcating Killing horizons are special classes of isolated horizons, such analysis would offer complementary insights into the properties of the latter horizons. The significance of isolated horizons to thermodynamics and black hole physics has been addressed in [16]. However, as we also have emphasized, the horizons constructed and discussed in this work are special in the sense that they have been generated using the Weyl formalism. An extension of the present work to the case of Einstein‘s gravity coupled to more general types of fields would require a different treatment than the one presented here [19] and such treatment is currently under way [25].

The results of the present work raise a number of questions that need further elaboration. We have not touched the issue of the thermodynamical properties of the distorted horizons. The two families of solutions presented in this work possess distinct thermodynamical properties and they will be discussed in a separate article.

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[4] For a relatively recent report on the status of Cosmic Censorship as well as properties and current understanding of dynamical spacetimes, see R. M. Wald, gr-qc/ 9710068.

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[18] It should be stressed here that the Einstein-Klein-Gordon system may admit solutions possessing Killing horizons of diverse topologies. Of particular importance would be solutions admitting an $\mathbb{R} \times S \times S$ horizon. Such solutions are associated with toroidal black holes and their constructions and properties will be discussed elsewhere: J. Estevez Delgado and T. Zannias, "On Toroidal Black Hole Solutions and Other Axisymmetric Solutions of Einstein-Klein-Gordon System" (to be published).

[19] It ought to be noticed here that the construction of the Weyl chart makes use of the field Eqs. (1) and (2). The fact that such a chart is admitted is traced in the condition $R_{\mu\nu}(\xi^{\mu}_{(t)}\xi^{\nu}_{(t)} + \xi^{\mu}_{(\varphi)}\xi^{\nu}_{(\varphi)}) = 0$ guaranteed by the massless nature of Φ and its invariance along the orbits of the Killing fields.

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[22] Notice that had we chosen to extend (g, Φ) maximally as a solution of the Einstein-Klein-Gordon system, then we could have taken $r \in (0, 2m + r_b)$. For such an extension the curvature of the solution (g, Φ) grows unbounded as $r \to 0$. No attempts have been made to analyze the structure of this singularity.

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